NUMERICAL TWO-DIMENSIONAL ANALYSIS OF A RING DF–CO_2 CONTINUOUS CHEMICAL LASER TAKING INTO ACCOUNT REAGENT MIXING

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UDC 621.373.826.038.823

It was pointed out in [1, 2] that the use of expanding supersonic flows with an appropriate choice of the level of initiation and degree of gasdynamic expansion enables conditions to be created under which the specific energy indicators of a continuous chemical DF–CO_2 laser are almost two times higher than the analogous indicators of a laser whose nozzle block has a flat geometry. Thus, calculations show that it is possible in principle to control the energy characteristics of a DF–CO_2 laser by gasdynamic methods. We note that the experimental realization of the conditions indicated in [1, 2] does not require a cylindrical nozzle block with an axially symmetrical flow (Fig. 1). In practice it is possible, for example, to use a sectoral model with a similar construction. It is only important that the flare angle of the sector be large enough. In this case, the edge effects distort the radial nature of the flow very little and the results obtained for a ring model of the laser remain in force.

The calculations in [1, 2] were performed on the basis of a one-dimensional model; reagent displacement effects were ignored. In the general case, in order to take these effects into account it is better to use self-consistent two-dimensional models, which enable a more accurate description of the process of mixing of the oxidizer and fuel flows.

In this paper, using a two-dimensional approach and taking into account viscosity, diffusion, and heat conduction, we study the effect of a finite rate of mixing on the specific energy indicators of DF–CO_2 lasers with flat and cylindrical nozzle blocks.

1. Basic Equations. Restricting ourselves to the analysis of an autonomous variant of a chemical DF–CO_2 laser, we recall (see also [1, 2]) that in such a laser the reaction

$$2CO + O_2 \rightarrow 2CO_2, \tag{1.1}$$

which provides laser CO_2 molecules without the formation of secondary products, is used as an auxiliary reaction in order to create active centers (fluorine atoms). Heat liberated in the course of the reaction is used to create active centers, which are formed by the thermal dissociation of molecular fluorine, mixed into the productions of the reaction (1.1): $F_2 + Q \rightarrow 2F$. The required degree of initiation (the starting number of fluorine atoms) is ensured by diluting a CO/O_2 mixture with an inert gas (helium), which controls the temperature of the combustion products.

The cylindrical nozzle block in the ring model consists of a collection of small coaxial ring-shaped nozzles, alternating in a definite order (see Fig. 1) for the oxidizer (F, F_2, CO_2, He) and fuel (D_2, CO_2, He) flows. At the outlet of the nozzle block the oxidizer and fuel flows are mixed with one another and as a result of the chain reaction

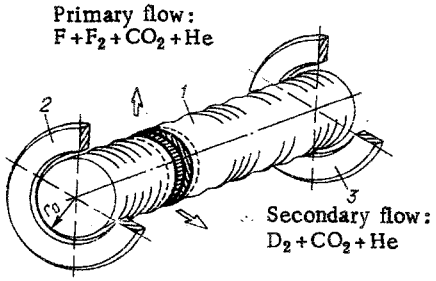

Fig. 1

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 3-9, July-August, 1985. Original article submitted May 30, 1984.

0021-8944/85/2604-0449$09.50

$$F + D_2 \rightarrow DF(V) + D,\ D + F_2 \rightarrow DF(V) + F,$$

vibrationally excited deuterium fluoride molecules are formed in the active medium of the laser. Later, as a result of the vibrational VV exchange, energy is transferred from the excited molecules into the antisymmetrical mode of CO_2 and a population inversion of CO_2 molecules, giving rise to lasing at the wavelength $\lambda = 10.6$ μm, is created on the transition of the band $00^01 \rightarrow 10^00$. We note that aside from the indicated processes, and entire chain of different chemical-kinetic transformations occurs in the active medium (see, for example, [1-4]).

Let h_1 and h_2 be the width of the oxidizer and fuel jets along the z axis, and let $h_* = (1/2)(h_1 + h_2)$ be the half-width of the period of the nozzle array. Usually, $h_* \ll \Delta r_{las}$ (Δr_{las} is the characteristic extent of the laser zone along the flow), and for this reason the motion of the gas in the region of the resonator is in many ways analogous to viscous flow in narrow channels. It is well known (see, for example, [5-7]) that the approximate equations of gasdynamics, describing viscous flow of a compressed gas in narrow channels, have the same form as the boundary-layer equations. Nevertheless, there is a fundamental difference between the quantitative description of the motion of a gas in the narrow-channel approximation and in the boundary-layer approximation.

Indeed, in a thin boundary layer the pressure distribution along the flow is known from the solution of the problem for an external nonviscous flow [8, 9]. In the case of viscous flow in a narrow channel, however, the pressure distribution along the flow is not known beforehand and must be determined from a self-consistent solution of all the equations of gasdynamics. It is precisely this fact that determines the difference in the methods used to solve the boundary-layer equations and the narrow-channel equations. It is also interesting to note that even in the special case of "free" expansion (with a flat nozzle and jets with low heights), when the gas pressure does not change along the flow, the flow conditions differ fundamentally from those of a flow in a boundary layer, since the degree of this expansion along the flow is unknown. As shown, for example in [10], the degree of expansion under such conditions is determined on the basis of the self-consistent solution of all the equations of gasdynamics.

Since in this paper we study only the calculated conditions of efflux from the nozzle, the transverse gradient of the pressure can be neglected below: $(\partial p/\partial z = 0)$. It is shown in [11, 12] that this approximation satisfactorily describes the process of mixing of the calculated jet flows of the chemically reacting gas even in the presence of intense heat liberation in the flow.

Based on the above, for a cylindrical (radial) flow of viscous reacting gas, the starting equations of gasdynamics in the narrow-channel approximation can be written in the form

$$\begin{gathered}
\frac{\partial}{\partial r}(\rho u r) + \frac{\partial}{\partial z}(\rho w r) = 0, \\
\rho u \frac{\partial u}{\partial r} + \rho w \frac{\partial u}{\partial z} = -\frac{dp}{dr} + \frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right), \\
\rho u \frac{\partial h}{\partial r} + \rho w \frac{\partial h}{\partial z} = u \frac{dp}{dr} + \mu\left(\frac{\partial u}{\partial z}\right)^2 + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) + \frac{\partial}{\partial z}\left(\rho \sum_i h_i D_i \frac{\partial C_i}{\partial z}\right) - gI, \\
p = \rho R T / W, \quad \rho u \frac{\partial C_i}{\partial r} + \rho w \frac{\partial C_i}{\partial z} = \dot{w}_i + \frac{\partial}{\partial z}\left(\rho D_i \frac{\partial C_i}{\partial z}\right), \\
\rho u \frac{\partial E_k}{\partial r} + \rho w \frac{\partial E_k}{\partial z} = \dot{w}_{E_k} + \frac{\partial}{\partial z}\left(\rho D_k \frac{\partial E_k}{\partial z}\right).
\end{gathered} \tag{1.2}$$

Here ρ is the density of the mixture; u and w are the radial and transverse components of the flow velocity; T is the temperature; p is the pressure; $h = \sum_i h_i C_i$ is the specific enthalpy of the mixture; h_i is the enthalpy of the i-th component; $C_i = \rho_i/\rho$ is the relative mass concentration of the i-th component; W is the molecular weight of the mixture; μ, λ, and D_i are the coefficients of dynamic viscosity, thermal conductivity and diffusion; $E_k = \sum_v v C_k(v)$ is the dimensionless vibrational energy of the k-th component; $\dot{w}_i$ and $\dot{w}_{E_k}$ are source terms, describing chemical, vibrational, and radiation kinetic processes; g is the

local gain on the standard transition of the P(20) band $00^01 \rightarrow 10^00$; I is the radiation intensity; and R is the universal gas constant.

Unlike the system of boundary-layer equations [when the pressure distribution p(r) along the flow is assumed to be known], the system (1.2) in the narrow-channel approximation is not closed, since the number of variables (ρ, u, w, p, T) exceed the number of equations [the equation $\partial p/\partial z = 0$ merely implies that p = p(r) and is independent of the coordinate z]. To close the system of narrow-channel equations it is necessary to use the condition that the flow rate over the width of the half-period of the nozzle structure be constant:

$$\int_0^{h_*} \rho u r dz = G_0 = \text{const}. \tag{1.3}$$

In addition, the system is supplemented by appropriate boundary conditions at the cutoff of the nozzle block ($r = r_0$) and at the symmetry planes ($z = 0, h_*$):

at $r = r_0$,

$$p = p_0, \quad u = u_0(z), \quad T = T_0(z), \quad C_i = C_i^0(z), \quad E_h = E_k^0(z);$$

at $z = 0, h_*$,

$$w = 0, \quad \frac{\partial u}{\partial z} = \frac{\partial T}{\partial z} = \frac{\partial C_i}{\partial z} = \frac{\partial E_h}{\partial z} = 0.$$

Using periodic boundary conditions and integrating the equation of continuity over a transverse cross section, we obtain the following expression for the transverse component of the velocity w:

$$\rho w = -\frac{1}{r}\int_0^z \frac{\partial}{\partial r}(\rho u r)\, dz. \tag{1.4}$$

Finally, using the equation of state and the condition that the flow rate (1.4) be constant, we find one more relation for the pressure:

$$p = G_0 R \Big/ \left[r \int_0^{h_*} \frac{uW}{T} dz \right]. \tag{1.5}$$

The relations (1.4) and (1.5), together with the equation of state and the equations of conservation of energy and momentum, form a closed system for integrating the equations of gasdynamics in the narrow-channel approximation. We note here that it is convenient to determine the pressure from the relation (1.5) and the density of the mixture from the equation of state.

The intensity of the radiation I in the lasing state is determined using the threshold condition

$$G = \frac{1}{h_*}\int_0^{h_*} g(r, z)\, dz = g_{\text{th}},$$

where G is the gain of the active medium averaged over the period of the structure and $g_{th} = L_a^{-1} \ln (r_1 r_2)^{-1/2}$ is the threshold gain (L_a is the length of the active medium, and r_1 and r_2 are the coefficients of reflection of the mirrors in the resonator).

The transport coefficients in a multicomponent mixture were represented as the sum of the laminar and turbulent components (labeled by the indices ℓ and t, respectively): $\mu = \mu_\ell + \mu_t$, $\lambda = \lambda_\ell + \lambda_t$, and $D_i = D^\ell_i + D^t$. Generally speaking, the experiments show that at high pressures the mixing of the jets at the nozzle cutoff is primarily of a turbulent nature, and, at the same time, $\mu_t \gg \mu_\ell$, $\lambda_t \gg \lambda_\ell$, and $D^t \gg D^\ell_i$.

The turbulent diffusion was described using the semiempirical theory of turbulence [8, 9, 13, 14]. According to Prandtl's hypothesis, the coefficient of turbulent viscosity μ_t in the one-parameter model of turbulence is defined by the relation

$$\mu_t = \rho l^2(r)|\partial u/\partial z|,$$

while the mixing length $l(r)$ for free jet flows can be represented in the form [14]

$$l(r) = b_l(r - r_0),$$

where $b_l = h/k$ (h is the characteristic period of the structure and k is a constant; usually, $k \cong 10$).

In reality, however, it is necessary to take into account the fact that although the turbulent mixing occurs, as a rule, very rapidly (over distances of ≅10 periods of the structure from the nozzle cutoff, the flows are virtually completely mixed); nevertheless, the mixing at the molecular level, associated with the decay of turbulent eddies, proceeds much more slowly. This must obviously be kept in mind when describing chemical and relaxation processes in the active medium. On the basis of the simplified semiempirical model adopted in this work, the indicated effect was taken into account by increasing the constant k in the definition of b_l by an order of magnitude (it was assumed that k = 100).

It is pointed out in [13] that, in the presence of turbulent mixing of jets, to a good approximation the turbulent analogs of the Schmidt number Sc_t and Prandtl number Pr_t can be set equal to $Sc_t = \mu_t/\rho D_t \cong 0.7$ and $Pr_t = \mu_t c_p/\lambda_t \cong 0.7$. The coefficients λ_t and D_t are immediately determines with the use of these two relations.

2. Brief Description of the Algorithm and Computational Results. Generally speaking, the narrow-channel equations (and, correspondingly, the boundary-layer equations) acquire a simpler form after the transformation to Mies variables [10, 15, 16]. Nevertheless, as pointed out for example in [17], under the conditions studied Mies variables turn out to be ineffective. The problem is that, in describing the mixing of jets with substantially different densities on the basis of Mies variables, in order to ensure satisfactory accuracy of the calculation a different grid with a variable step in the transverse direction must be introduced. This, as is well known, lowers the order of the approximation of the starting equations of gasdynamics.

Because of this, in this paper the problem is solved in the coordinates (r, z). Numerical integration is carried out using the difference scheme examined in [17] (with some alterations). In addition, in order to determine the intensity of the radiation under conditions of stationary lasing, the approach first proposed in [17] is also used. We note that, in contrast to the well-known traditional method [3, 4], in this approach the intensity of the radiation with a large integration step is determined with a much higher accuracy.

The equations of kinetics, the equation of motion, and the equation for the temperature of the mixture are of the parabolic type and can therefore be integrated by the "marching" method. A two-layer implicit iterative scheme of second-order accuracy (a Crank–Nicholson type scheme) is used to form their difference approximation. In contrast to [17], in order to raise the stability of the scheme the convective terms of these equations were approximated with the help of a difference scheme for numerical differentiation "against the flow" [18].

In a concrete implementation of the iteration process all variables in the (n + 1)-th layer along r were assigned in the first iteration the values of the corresponding quantities from the n-th layer. The iteration continued until the integrated quantities converged with a fixed relative accuracy (about 10^{-3}). The integration step along the longitudinal coordinate (in the transverse direction 20 steps in the difference grid were used) was chosen primarily in accordance with the requirement that the required accuracy of integration of the kinetic equations be ensured.

As a control, the results of the two-dimensional calculations were compared for the same starting data with the results obtained previously based on the one-dimensional model. The comparison showed satisfactory agreement (within ≅5%).

In performing specific calculations, the parameters of the oxidizer and fuel flows, the half-period of the nozzle block structure ($h_* = 0.3$ cm), and the ratio of the jet width ($h_1/h_2 = 1$) were given as the starting data. The pressure in the jets was assumed to be the same, and the value of the threshold gain was assumed to be equal to $g_{th} = 1.25 \cdot 10^{-3}$ cm^{-1} (this corresponds to the value used in one-dimensional calculations [1, 2]).

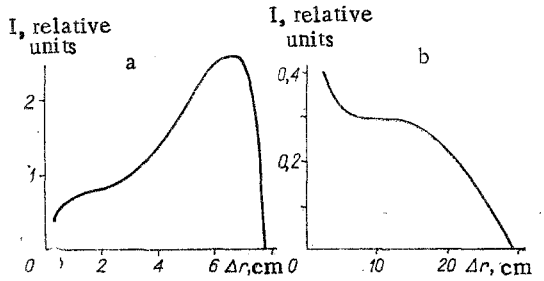

Fig. 2

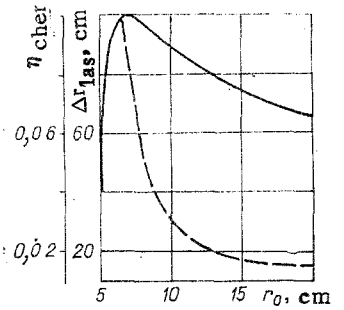

Fig. 3

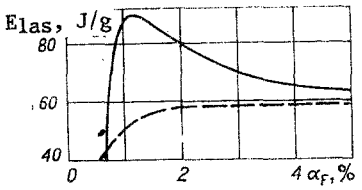

Fig. 4

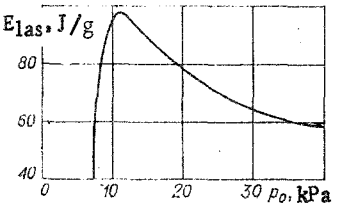

Fig. 5

Using the results from the optimization of the composition of the mixture [2], the following flow parameters at the cutoff were used:

For the oxidizer flow the flow temperature was T_1 = 300°K, the flow velocity was u_1 = 2 km/sec, the composition of the mixture was $F + F_2:CO_2:He = 1:4:10$, and the degree of dissociation of molecular fluorine α_F was varied in the calculations;

for the fuel flow the temperature was T_2 = 200°K, the flow velocity was u_2 = 1.5 km/sec, and the composition of the mixture was $D_2:CO_2:He = 1:4:10$.

The calculations performed in principle confirmed the validity of the basic results obtained in [1, 2]. Nevertheless, the two-dimensional analysis revealed some additional peculiarities of the system under study.

Figure 2a, b shows the typical distributions of the radiation intensity for the flat and cylindrical (the nozzle radius r_0 = 10 cm) geometries with α_F = 1% and p_0 = 13.3 kPa. The case of a flat nozzle block was modeled in the calculations by a cylindrical nozzle with the radius r_0 = 10 m.

Analysis shows that when mixing effects are taken into account, the width of the laser zone in the flat case increases; in the cylindrical case, on the contrary, it decreases by approximately 20-30%. The shortening of the laser zone in the cylindrical case (owing to the effect of nonuniformities on the amplifying properties of the medium) is also accompanied by some lowering of the energy indicators of the laser as compared with [1, 2]; in the flat case an increase in the width of the lasing zone, for all practical purposes, does not increase the energetics of the laser (to within 1-2%).

Figure 3 shows the dependence of the chemical efficiency (solid line) and the width of the laser zone (broken line) of the DF–CO_2 laser on the radius of the nozzle r_0 (for α_F = 1% and p_0 = 13.3 kPa). It is evident that for r_0 = 10 cm, the efficiency of the laser is 1.5 times higher, and not two times higher (as predicted in [1, 2]) than in the flat case. And, although an even larger advantage can be achieved with the smaller values of r_0, it is evident from Fig. 3 that the width of the laser zone becomes unreasonably large at the same time.

Figure 4 shows the dependence of the specific laser energy on the initial level of initiation α_F for the flat (broken line) and cylindrical (solid line, r_0 = 10 cm) geometries with p_0 = 13.3 kPa. It is interesting to note that unlike [1, 2], the two-dimensional approach predicts that the energy indicators of the laser are somewhat less sensitive to the level of initiation α_F. The latter fact can in principle be significant when optimizing the ring model.

Finally, as in the case of the one-dimensional analysis, the two-dimensional calculations also show the existence of an optimum with respect to the starting pressure at the cut-

off of the nozzle under the conditions of the cylindrical construction (for low values of αF). Figure 5 shows the dependence of the specific laser energy on the magnitude of the pressure at the cutoff (with αF = 1%, r_0 = 10 cm). Compared with the one-dimensional approach, the only difference here is that, in the two-dimensional case, the magnitude of the optimal pressure decreases somewhat (from 13.3 to 10.7 kPa), which is explained primarily by the same factors which were mentioned above. At high pressures ($p_0 \cong$ 40-50 kPa), the energy indicators of the ring variant of the DF–CO_2 laser drop appreicably and approach the typical characteristics of a laser with a flat nozzle. This is attributable to the shortening of the laser zone when the pressure is increased, as a result of which the nature of the geometry is no longer important.

In conclusion, we note that the main purpose of this work was to determine the energy indicators of the ring model of a supersonic chemical DF–CO_2 laser. Numerous calculations have shown that, in this case, the use of a narrow-channel model is entirely justified. To obtain more subtle information, associated with the distribution of the gasdynamic parameters in the lasing zone (weak shocks and rarefaction waves, return flows in the bottom region, etc.), more complicated models based on the complete system of Navier–Stokes equations are used (see, for example, [19, 20]). Here, however, it should be kept in mind that the real flow pattern evidently depends substantially on the initial parameters of the chemically active jets being mixed (in particular, on the degree of turbulence), whose correct determination is in itself by no means a trivial problem.

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